MATHS

Part-A:

① $R = \{(-5,-6),(-6,-5),(-5,-5)\}$

② The binary operation can be defined as operation * which is performed on two elements from set A. The result of the operation is another element from same set A.

③ $y = \cos^{-1}\left(-\frac{1}{2}\right)$

We know that $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\therefore$

$y = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore$ principal value $= \frac{2\pi}{3}$

④ $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$\sin^{-1}1 = \sin^{-1}\frac{1}{5} + \cos^{-1}x$

$\frac{\pi}{2} = \sin^{-1}\frac{1}{5} + \cos^{-1}x$ $\qquad \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \quad x = \frac{1}{5}$

⑤ A matrix is said to be a row matrix if it has only one row. In general $B = [b_{ij}]_{1\times n}$ is a row matrix of order $1\times n$.

⑥ $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$= x^2 - \cancel{36} = 36 - \cancel{36}$

$\Rightarrow x^2 = 36$

$x = \pm 6$

⑦ $y = e^{\cos x}$

$\frac{dy}{dx} = e^{\cos x}(-\sin x) = -\sin x\, e^{\cos x}$

(8) $y = \sin(x^2+5)$

$\frac{dy}{dx} = \cos(x^2+5) \times (2x) = 2x\cos(x^2+5)$

(9) $\int (2x^2 + e^x)\,dx = \frac{2x^3}{3} + e^x$

(15) $P(B) = 0.5 \quad P(A \cap B) = 0.32$

$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.32}{0.5} = 0.64$

Part - B!

(17). $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \frac{\pi}{3} - (\pi - \sec^{-1}(2))$ $\qquad \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$= \frac{\pi}{3} - \pi + \frac{\pi}{3}$

$= \frac{2\pi}{3} - \pi = \frac{2\pi - 3\pi}{3} = \frac{-\pi}{3}$

(18) $y = \tan^{-1}x$

Domain is R

Range is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(19) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

From equality of matrices,

$y = 4$, $z = 3$, $x = 1$

(20). Let (x, y) be a point on a line joining $(1,2)$ and $(3,6)$

Thus all points lie on a same line.

$\therefore$ They do not form triangle.

$\therefore$ Area of triangle $= 0$

$\because$ Area is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x(2-6) - y(1-3) + (6-6) = 0$$

$$\Rightarrow -4x + 2y = 0$$

$$\Rightarrow 4x = 2y$$

$$y = 2x$$

(21). $x^2 + xy + y^2 = 100$

Differentiate wrt x.

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(x + 2y) = -(2x + y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$$

(22). $x = at^2 \qquad y = 2at$

$$\frac{dx}{dt} = 2at \qquad \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{\cancel{2a}}{2at} = \frac{1}{t}$$

(23). let $y = \sin(\cos(x^2))$

$\frac{dy}{dx} = \cos(\cos(x^2)) \times (-\sin(x^2)) \times 2x$

$= -2x \times \cos(\cos(x^2)) \times \sin(x^2)$

(24). $y = x^3 - x + 1$

Slope of tangent is $\frac{dy}{dx}$.

$\frac{dy}{dx} = 3x^2 - 1$

tangent whose x-coordinate is 2 is $\left.\frac{dy}{dx}\right|_{x=2}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = 3(2^2) - 1$

$= 12 - 1 = 11$

$\therefore$ Slope of tanget is $\underline{\underline{11}}$.

(25). $\int \frac{(\log x)^2}{x} dx$

let $\log x = t$

$\Rightarrow \frac{1}{x} dx = dt$

$\therefore \int t^2 dt = \frac{t^3}{3}$

$= \frac{[\log x]^3}{3}$

(26) $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x}$

$$= \int \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \int \sec^2 x - \operatorname{cosec}^2 x$$

$$= \tan x + \cot x + C$$

(27). $\int_0^{\pi/2} \cos^2 x \, dx$

$$= \int_0^{\pi/2} \frac{1+\cos 2x}{2}$$

$$= \frac{1}{2}\int_0^{\pi/2} 1 \, dx + \frac{1}{2}\int_0^{\pi/2} \cos 2x \, dx$$

$$= \frac{1}{2}[x]_0^{\pi/2} + \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$= \frac{\pi}{4} + \frac{1}{4}[1-1] = \frac{\pi}{4}$$

Part-C:

(34). $R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$;

$\because$ $(1,1), (2,2)$ and $(3,3) \in R$
$\therefore R$ is reflexive.

$(1,2) \in R$ but $(2,1) \notin R$
$\therefore R$ is not symmetric.

$(1,2) \in R$, $(2,3) \in R$ but $(1,3) \notin R$
$\therefore R$ is not transitive.

(35). $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

LHS:

$$2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} \rightarrow (1)$$

We know that

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore\ 2\tan^{-1}\frac{1}{2} = \tan^{-1}\left(\frac{2\times 1/2}{1-(1/2)^2}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

$\therefore$ (1) becomes

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\frac{31}{17} = RHS$$

Hence proved.

(36) Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

we know that

$$A = IA$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow \frac{1}{2}R_1$$

$$\begin{bmatrix} 1 & 3/2 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 5R_1$$

$$\begin{bmatrix} 1 & 3/2 \\ 0 & -1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -5/2 & 1 \end{bmatrix} A$$

$R_2 \to -2R_1$

$$\begin{bmatrix} 1 & 3/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 5 & -2 \end{bmatrix} A$$

$R_1 \to R_1 - \frac{3}{2} R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$$

This is similar to

$$I = A^{-1}A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

(38). $xy = e^{x-y}$

Taking log on both sides

$\log x + \log y = \log(e^{x-y})$

$= \log x + \log y = x - y$

Differentiate w.r.t x.

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} + 1\right) = 1 - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$

(39). $x = a(\cos\theta + \theta\sin\theta)$ $\quad$ $y = a(\sin\theta - \theta\cos\theta)$

$$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta) = a\theta\sin\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{a\theta}\sin\theta}{\cancel{a\theta}\cos\theta} = \tan\theta$$

(40). $f(x) = x^2 - 4x - 3$ $x \in [a, b]$ where $a = 1$, $b = 4$

Conditions of mean value theorem

1) $f(x)$ is continuous at (a, b)

2) $f(x)$ is derivable at (a, b)

Then there exists c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Thus for $f(x) = x^2 - 4x - 3$

Here $f(x)$ is a polynomial function and every polynomial function is continuous.

$\therefore$ $f(x)$ is continuous on $x \in [1, 4]$.

$f(x)$ is a polynomial and every polynomial function is differentiable. Thus $f(x)$ is differentiable at $x \in [1, 4]$.

$$f(x) = x^2 - 4x - 3$$
$$f'(x) = 2x - 4$$
$$\Rightarrow f'(c) = 2c - 4$$
$$f(a) = f(1) = 1 - 4 - 3 = -6$$
$$f(b) = f(4) = 4^2 - 16 - 3 = -3$$

$\therefore$ By mean value Theorem,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c - 4 = \frac{-3 + 6}{4 - 1} = \frac{3}{3} = 1$$

$$\Rightarrow 2c = 5$$

$$c = 5/2$$

(42) $\int \frac{xe^x}{(1+x)^2} dx$

$= \int e^x \left(\frac{x}{(1+x)^2}\right) dx$

$= \int e^x \left(\frac{1+x-1}{(1+x)^2}\right) dx$

$= \int e^x \left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right) dx$

let $f(x) = \frac{1}{1+x}$ $\therefore f'(x) = \frac{-1}{(1+x)^2}$

so, we get

$\int \frac{xe^x}{(1+x)^2} dx = \int e^x (f(x) + f'(x)) dx$

We know that,

$\int e^x (f(x) + f'(x)) dx = e^x f(x) + c.$

$\therefore$ we get

$\int \frac{xe^x}{(1+x)^2} = \frac{e^x}{1+x} + c.$

(43) $\int \frac{1}{(x+1)(x+2)} dx$

$= \int \frac{(x+2)-(x+1)}{(x+1)(x+2)} dx$

$= \int \frac{1}{x+1} - \frac{1}{x+2} dx$

$$= \int \frac{1}{x+1}\,dx - \int \frac{1}{x+2}\,dx$$

$$= \log(x+1) - \log(x+2) + C$$

$$= \log\left(\frac{x+1}{x+2}\right) + C.$$

(48) Given $\vec{a}, \vec{b}, \vec{c}$ are unit vectors,

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$|\vec{a} + \vec{b} + \vec{c}| = 0$$

Now,

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$= (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$$

$$= \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c}$$

using $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

$$= \vec{a}.\vec{a} + \vec{b}.\vec{b} + \vec{c}.\vec{c} + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$$

using $\vec{a}.\vec{a} = |\vec{a}|^2$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$$

$$0 = 1 + 1 + 1 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$$

$$-3 = 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$$

$$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-3}{2}$$

(51). S_1: man speaks truth

S_2: man lies

E: six on dice.

we need to find probability that is actually a six, if the man reports that it a six.

i.e

$$P(S_1/E)$$

$$P(S_1/E) = \frac{P(S_1) \cdot P(E/S_1)}{P(S_1) \cdot P(E/S_1) + P(S_2) \cdot P(E/S_2)}$$

$P(S_1) = \frac{3}{4}$ $\quad P(S_2) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(E/S_1)$ = Probability that six appears on dice, if the man speaks truth $= \frac{1}{6}$

$P(E/S_2)$ = Probability that six appears on dice, if the man lies

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

$$\therefore P(S_1/E) = \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}}$$

$$= \frac{3}{3+5} = \frac{3}{8}$$

Part - D:

52. Given: $A = R - \{3\}$ and $B = R - \{1\}$

$f: A \to B$ is defined by $f(x) = \frac{x-2}{x-3}$

$f(x_1) = f(x_2) \ \forall x_1, x_2 \in A$

i.e

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$x_1\cancel{x_2} - 3x_1 - 2x_2 + \cancel{6} = x_1\cancel{x_2} - 2x_1 - 3x_2 + \cancel{6}$$

$$-3x_1 + 2x_1 = -3x_2 + 2x_2$$

$$-x_1 = -x_2$$

$$\Rightarrow x_1 = x_2$$

$\therefore f$ is one-one.

Let $y \in B$ (Co-domain) be any element such that its pre-image is x.

$$f(x) = y$$

$$\frac{x-2}{x-3} = y$$

$$x - 2 = xy - 3y$$

$$3y - 2 = xy - x$$

$$3y - 2 = x(y-1)$$

$$x = \frac{3y-2}{y-1} \in A$$

$\therefore f$ is onto.

Hence f is bijective function.

(54). $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

(i) $(A+B) = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \rightarrow (1)$$

$$A^T + B^T = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \rightarrow (2)$$

From (1) and (2)

$$\underline{(A+B)^T = A^T + B^T}$$

(ii) $$A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$(A-B)^T = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \rightarrow (3)$$

$$A^T - B^T = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \rightarrow (4)$$

From (3) and (4)

$(A-B)^T = A^T - B^T$

(55). $2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}_{3\times 3} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1} \quad B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}_{3\times 1}$$

i.e

$AX = B$

$X = A^{-1}B$ where $A^{-1} = \dfrac{\text{adj} A}{|A|}$, $|A| \neq 0$

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

$|A| = 2(4+1) - 3(-2-3) + 3(-1+6)$

$= 10 + 15 + 15$

$= 40 \neq 0$

$a_{11} = 2$, $a_{12} = 3$, $a_{13} = 3$

$a_{21} = 1$, $a_{22} = -2$, $a_{23} = 1$

$a_{31} = 3$, $a_{32} = -1$, $a_{33} = -2$

$M_{11} = 4 + 1 = 5$; $A_{11} = (-1)^{1+1}(5) = 5$

$M_{12} = -2 - 3 = -5$; $A_{12} = (-1)^{1+2}(-5) = 5$

$M_{13} = -1 + 6 = 5$; $A_{13} = (-1)^{1+3}(5) = 5$

$M_{21} = -6+3 = -3$; $A_{21} = (-1)^{2+1}(-3) = 3$

$M_{22} = -4-9 = -13$; $A_{22} = (-1)^{2+2}(-13) = -13$

$M_{23} = -2-9 = -11$; $A_{23} = (-1)^{2+3}(-11) = 11$

$M_{31} = 3+6 = 9$; $A_{31} = (-1)^{3+1}(9) = 9$

$M_{32} = 2-3 = -1$; $A_{32} = (-1)^{3+2}(-1) = 1$

$M_{33} = -4-3 = -7$; $A_{33} = (-1)^{3+3}(-7) = -7$

$\therefore$ cofactor matrix of A $= \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}_{3\times 3}$

$\therefore \text{adj } A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}_{3\times 3}$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}_{3\times 3}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}_{3\times 1}$$

$$= \frac{1}{40} \quad \cancel{25=1} \quad \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\Rightarrow \underline{\underline{x=1}}, \quad \underline{\underline{y=2}}, \quad \underline{\underline{z=-1}}$

(56). $y = 3\cos(\log x) + 4\sin(\log x)$

differentiating wrt x

$$y_1 = -\frac{3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$

$$xy_1 = -3\sin(\log x) + 4\cos(\log x)$$

differentiating wrt x

$$y_1 + xy_2 = \frac{-3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$

$$y_1 + xy_2 = \frac{-1}{x}\left(-3\cos(\log x) + 4\sin(\log x)\right)$$

$$y_1 + xy_2 = \frac{-1}{x}(y)$$

$$\Rightarrow xy_1 + x^2y_2 = -y$$

$$\Rightarrow xy_1 + x^2y_2 + y = 0$$

(57). Let length of rectangle $= x$
width of rectangle $= y$

$\frac{dx}{dt} = -5$ cm/min $\rightarrow$ ① $\rightarrow$ length is decreasing at the rate of 5 cm/min

width is increasing at the rate of 4 cm/min

$\frac{dy}{dt} = 4$ cm/min

a) P $\rightarrow$ Perimeter

$P = 2(x+y)$

$$\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}.$$

$$= 2(-5+4) = -2$$

$$\therefore \frac{dP}{dt} = -2 \text{ cm/min}$$

$\therefore$ Perimeter is decreasing at the rate of 2 cm/min.

(b) Let A be the area of rectangle.

Area $= A = x \times y$

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dA}{dt} = (-5)y + 4(x)$$

$$\left.\frac{dA}{dt}\right|_{(8,6)} = 4(8) - 5(6)$$

$x=8,\ y=6$

$$= 32 - 30$$

$$\Rightarrow \frac{dA}{dt} = 2 \text{ cm}^2/\text{min}$$

$\therefore$ Area is increasing at the rate of $2 \text{ cm}^2/\text{min}$.

(63) Let E : A solves the problem

F : B solves the problem

Since E & F are independent events

$P(E \cap F) = P(E) \cdot P(F)$ $\qquad P(E) = \frac{1}{2},\ P(F) = \frac{1}{3}$

$$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

(i) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$

$= \frac{5-1}{6} = \frac{4}{6} = \frac{2}{3}$

(ii) $[P(E \cap F') \cup P(E' \cap F)]$

$E - F \qquad F - E$

$E \cap F' \cup F \cap E'$

$= P(E \cap F') + P(E' \cap F)$

E & F are independent,
Thus E' & F , E & F' are also independent.

$\therefore = P(E) \cdot P(F') + P(E') \cdot P(F)$

$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \left(1 - \frac{1}{2}\right)\frac{1}{3}$

$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} + \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

Part - E :

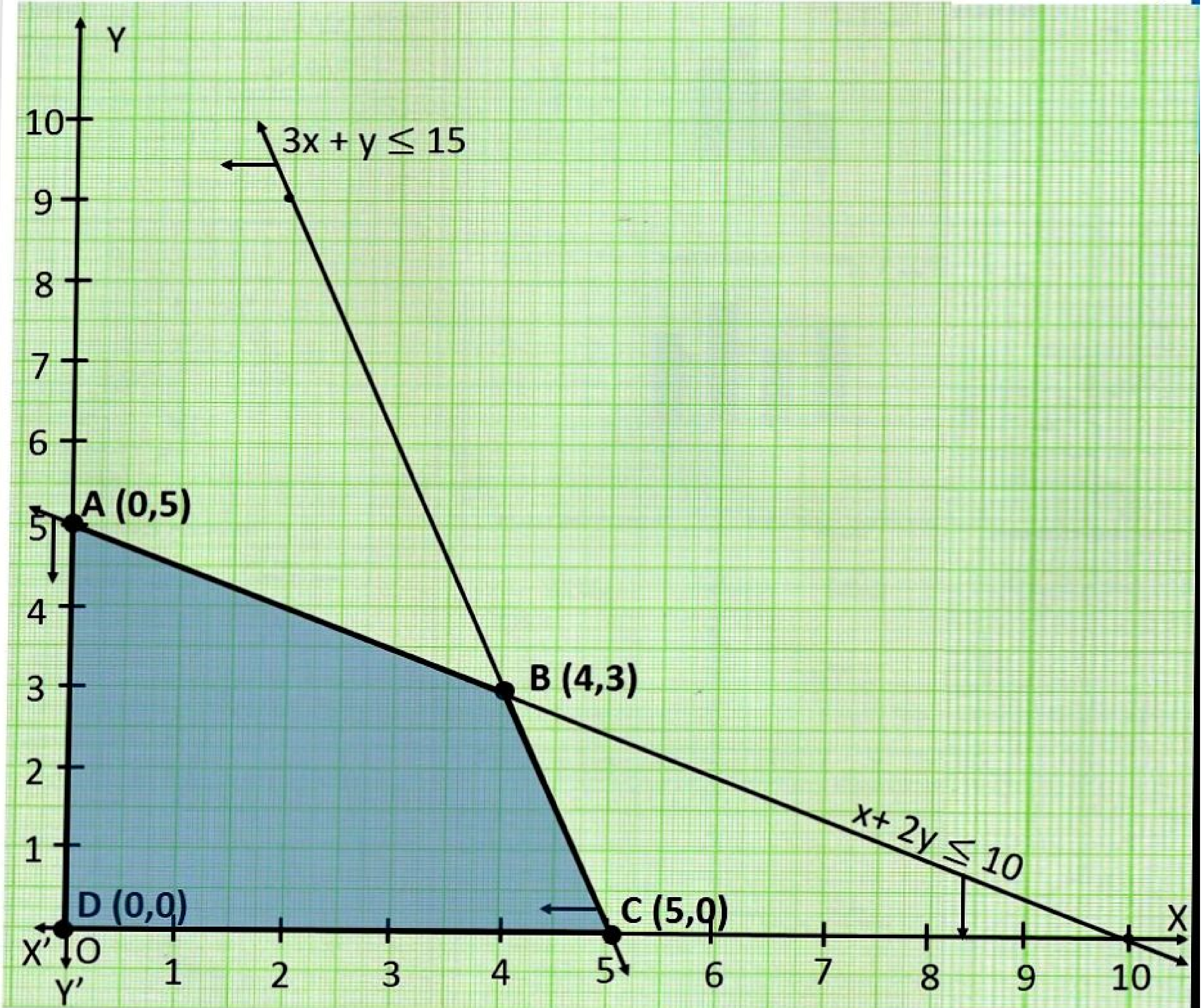
(64) (a) Maximise $z = 3x + 2y$
Subjected to
$x + 2y \leq 10$
$3x + y \leq 15$
$x, y \geq 0$

$x + 2y \leq 10$

x	0	10
y	5	0

$3x + y \leq 15$

x	0	5
y	15	0



Y
3x + y ≤ 15
A (0,5)
B (4,3)
x+ 2y ≤ 10
D (0,0)
C (5,0)
X
X'
O
Y'
1 2 3 4 5 6 7 8 9 10

Corner points	Value of z
(5, 0)	15
(4, 3)	18 → maximum
(0, 5)	10
(0, 0)	0

$\therefore Z = 18$ is maximum at (4, 3)

(B) Given: $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}_{2\times 2}$

To prove : $A^2 - 4A + I = 0$

$LHS = A^2 - 4A + I$

$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 0 = RHS$

To find A^{-1}:

$A^2 - 4A + I = 0$

$A^2 - 4A = -I$

Pre-multiply by A^{-1} on both sides,

$A^{-1}(A^2 - 4A) = A^{-1}(-I)$

$\underline{A^{-1}A}.A - 4\underline{A^{-1}.A} = -A^{-1}I$

$I.A - 4I = -A^{-1}$

$A - 4I = -A^{-1}$

$\Rightarrow A^{-1} = 4I - A$

$$A^{-1} = 4\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix} - \begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}_{2\times 2}$$